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BULLETIN

Calculating Freezing Times

This Bulletin describes how to calculate the freezing times of carcasses, sides and cartoned meat using commonly available spreadsheet tools and methods developed by MIRINZ researchers.

Freezing is the process of removing heat so that the water content of meat is converted into ice.

When liquid water changes to a solid (ice), a large amount of heat must be removed in addition to the amount that is removed to change the temperature. This extra amount is called the *latent heat of freezing*.

When a pure substance (like water) freezes, its temperature does not change during the time that its latent heat is being removed. Meat, however, is not a pure substance, so when it freezes the latent heat is not all removed at one fixed temperature. Nevertheless, quite a lot of heat will be removed before the temperature starts to fall again by very much, as shown in Figure 1.

Eventually, most of the water is frozen and the meat begins to simply cool to its storage temperature. This period of cooling after freezing is termed the sub-cooling stage.

THE FREEZING FRONT

A piece of meat starts to freeze when its surface temperature falls to the meat's freezing temperature, usually around minus 0.9°C (-0.9°C). The meat will then freeze from the outside in. In other words, the surface freezes first, then the frozen layer becomes thicker over time.

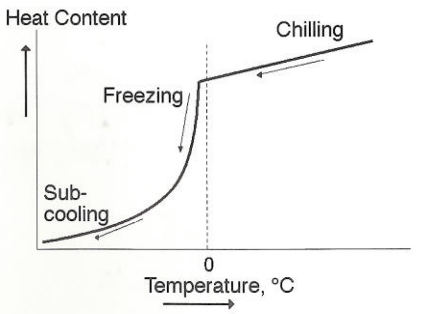

Figure 1. Variation in the heat content (enthalpy) of meat with temperature.

The boundary between the outer, frozen layer and inner, unfrozen meat is called the *freezing front*. This freezing front gradually moves inwards towards the centre of the meat, as shown in Figure 2.

A very short time after the freezing front has begun moving, the temperature of the unfrozen region has usually dropped to the freezing temperature of the meat. Therefore the temperature at the centre of the meat remains constant for most of the freezing process. This relatively constant temperature is sometimes called the plateau temperature, and is shown in Figure 3.

One consequence of the temperature profile shown in Figure 3 is that temperature measurements at the centre of a piece of meat will not indicate how well frozen the meat is, unless the meat is almost completely frozen and the centre temperature has begun to fall below the freezing temperature.

If the temperature measurement is not actually at the thermal centre, the plateau temperature measured at that site will still be the freezing temperature of the meat. However, the temperature at this point will start to drop from the plateau sooner than the temperature at the thermal centre, and the rate at which it drops will be slower.

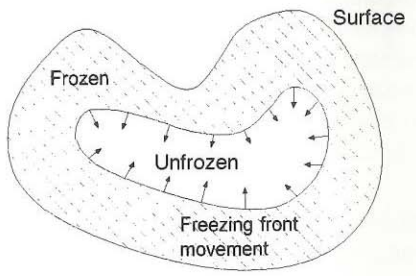

Figure 2. The freezing front.

The temperature fall at a non-central point is slower than at the centre because not only is the meat at the non-central point releasing its own heat content outwards into the rest of the meat, but other heat is also passing through it from the more central meat tissues.

As a consequence, the closer the temperature measurement is to the centre of the meat, the sharper is the "knee" in the temperature plot. For example, the curve in Figure 3 has a sharp "knee".

The bottom part of the curve in Figure 3 is "tailing out". This indicates that the meat temperature at the centre is starting to get close to that of the refrigerated medium (for example, air). (The reduced temperature difference slows the rate of cooling and therefore the rate of temperature fall.)

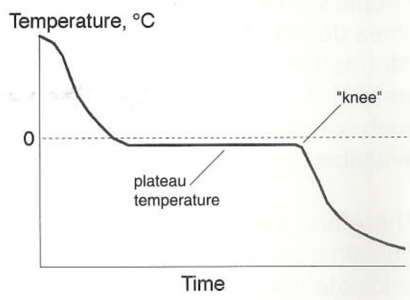

Figure 3. Meat centre temperature during freezing.

FREEZING TIME CALCULATION

When engineers design, modify or analyse a freezing process, they often need to predict the time required to freeze the product. A variety of calculation methods can be used to carry out such predictions.

The "Freezing Wheel"

MIRINZ developed a freezing time calculator, commonly known as the "freezing wheel", over 20 years ago. It consists of several concentric discs marked on their outside edges with product sizes, air temperatures and velocities, final product temperatures, and so on. Aligning the various discs to the selected freezing conditions moves an arrow on one of the discs to point at the predicted freezing time.

While it was quite ingenious, the freezing wheel has two important disadvantages: a different freezing wheel is required for each product type, and the freezing wheel is applicable only to the freezing conditions that its creators included during its design (for example, there is little choice of final temperature).

These problems make it impractical to use the freezing wheel except for common meat industry products and common conditions. In addition, the process of aligning the discs introduces some error.

Computer Software

Sophisticated computer software lies at the other extreme. Such software can simulate a variety of freezing processes and, for these, predict the freezing times of almost any product type, as well as chilling times, heat loads, microbial growth, and other outputs. Software to do this is available from MIRINZ.

Mathematical Model

People wanting to calculate freezing times do not always need the other information that freezing time computer software can provide, and such software may not always be available.

Therefore, the remainder of this Bulletin describes a another way to calculate freezing times.

Although the calculations given in this Bulletin look daunting, they can be carried out with any pocket calculator or can be conveniently set up in a spreadsheet program.

FREEZING TIME EQUATION

The theory behind the equation used to predict the freezing time is as follows:

Inside the freezing front, the meat is unfrozen, but at its freezing temperature; outside the freezing front, the meat is already frozen and its temperature is gradually dropping towards the temperature of the surroundings. As already said, during freezing, most of the heat to be removed is latent heat. This is released from a given part of the meat as the freezing front passes through it.

Therefore, the task of predicting the freezing time is simplified to the task of predicting how long it will take for the freezing front to get from the surface of the meat to the centre, and then adding factors to account for the initial chilling period and the final sub-cooling period.

Because both of these periods are usually relatively short compared to the freezing period, as can be seen from Figure 3, their lengths need not be predicted very accurately as long as the length of the freezing period is well-predicted.

In 1986, MIRINZ's Dr Tuan Pham developed the following equation to predict freezing time t_f:

$$t_f = \frac{1}{E}\left(\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right)\left(\frac{X}{h} + \frac{X^2}{2k_f}\right)$$

This is the freezing time equation.

Predicted freezing times calculated with this equation can be expected to be accurate within ±15%, although it is usually a bit better than that.

For the freezing time equation:

$$\Delta H_1 = \rho_l C_l (T_{in} - T_{fm})$$

$$\Delta H_2 = \rho_l L + \rho_f C_f (T_{fm} - T_c)$$

$$\Delta T_1 = \frac{T_{in} - T_{fm}}{2} - T_a$$

$$\Delta T_2 = T_{fm} - T_a$$

$$T_{fm} = 1.8 + 0.263\, T_c + 0.105\, T_a$$

Where:

- t_f is the freezing time, s
- E is the Equivalent Heat Transfer Dimensionality
- X is the shortest distance from the thermal centre to the surface, m
- h is the surface heat transfer coefficient (possibly including packaging resistance), W/m^2 K
- k_f is the frozen meat thermal conductivity, W/m K
- ΔH_1 is the heat released during chilling, J/m^3
- ΔH_2 is the heat released during freezing and subcooling, J/m^3
- ΔT_1 is the temperature driving force during chilling, °C
- ΔT_2 is the temperature driving force during freezing and subcooling, °C
- ρ_l, ρ_f are the densities of unfrozen and frozen meat, respectively, kg/m^3
- C_l, C_f are the specific heat capacities of unfrozen and frozen meat, respectively, J/kg K
- T_{in} is the initial temperature of the product, °C
- T_{fm} is the mean freezing temperature of the product, °C
- T_c is the final centre temperature of the product, °C
- T_a is the temperature of the cooling medium (e.g. air), °C
- L is the latent heat of freezing, J/kg

CALCULATING THE FACTORS FOR THE FREEZING TIME EQUATION

MIRINZ has measured the values of C_l, C_f, k_f, ρ_l and ρ_f for many meat industry products and these are given in MIRINZ Technical Report No. 955. [Note that the symbols used in that report are a little different. For example, k_f in that report is the thermal conductivity of the food at its freezing temperature, whereas here that symbol refers to the thermal conductivity of the food when thoroughly frozen (e.g. the thermal conductivity at T_c).]

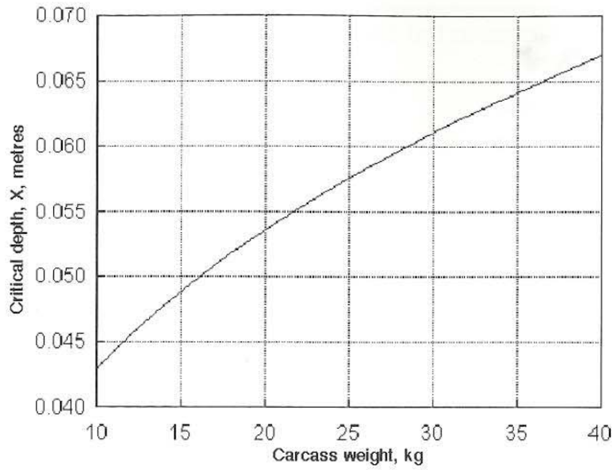

Figure 4. Deep leg depths for various lamb and sheep carcass weights.

Value of *L* (Latent Heat of Freezing)

The value of *L* is calculated as follows:

$$L = H_f - C_f (T_f + 40)$$

Where:

H_f is the heat content of the meat at the freezing temperature, J/kg

T_f is the actual freezing temperature of the meat, °C (usually about minus 0.9°C)

Value of *E* (Equivalent Heat Transfer Dimensionality)

The following table gives values for *E* (Equivalent Heat Transfer Dimensionality).

E was developed by researchers at Massey University to relate the cooling time for a complicated shape to the cooling time for a simple shape like a slab or a sphere.

Table 1. *E* values for products of interest to the meat industry.

Product	*E*
Lamb (shoulder)	1.4
Lamb (deep leg)	2.2
Ewe (deep leg)	2.0
Beef side or quarter (deep leg)	1.3
Beef carton (plate freezer)	1.0
Beef carton (air blast freezer)	1.3 to 1.5

For beef cartons, a range is shown. In this case, the exact value of *E* depends on the precise shape. In general, the thicker the object is relative to its width and length, the larger the *E* value is.

This means that a deep carton will have an *E* value of 1.5 whereas a shallow carton will have an *E* value of 1.3.

Value of *X* (Depth of Thermal Centre)

The parameter *X* is the "depth" of the thermal centre. For a carton, it is easy to calculate *X*, as it is simply half the carton thickness. For example, if a carton is 160 mm thick, then $X = 0.08$ m. For a carcass or side, however, it's a bit more complicated, and the *X* value can be read off a graph. Figure 4 is such a graph, for lamb and sheep carcasses.

For lamb and sheep carcasses, the value of *X* can be more precisely calculated by using the equation,

$$X = 0.002 + 0.019\sqrt[3]{m}$$

where *m* is the carcass weight in kg.

Value of *h* (Heat Transfer Coefficient) — Unpackaged Meat

We need the heat transfer coefficient *h* at the meat surface. This depends on the freezing medium – air, water, etc.

For air freezing, the surface heat transfer coefficient, h_{air}, in forced convection can be estimated from the following equations, developed by researchers at Massey University:

$$h_{air} = 7.3\, v^{0.8} \text{ W/m}^2\text{ K}$$
for flat surfaces

$$h_{air} = 12.5\, v^{0.6} \text{ W/m}^2\text{ K}$$
for curved surfaces

Where:

v is the air velocity in m/s.

h_{air} is the surface heat transfer coefficient in air.

The heat transfer coefficients predicted by these equations are shown in Figure 5. The flat surface equation should be used for cartoned product; the equation for curved surfaces should be used for lamb and sheep carcasses.

These equations are valid for air velocities of 0.4 m/s or higher. Below this air velocity, natural convection starts to significantly influence the heat transfer coefficient.

The heat transfer coefficient can be predicted for air velocities below 0.4 m/s, but the equations to do this are complicated. One can usually safely assume that the heat transfer coefficient at the surface of a meat product due to natural convection is approximately equal to the value shown at the lower end of the relevant line in Figure 5.

Equations for estimating both the air (natural convection) and water heat transfer coefficients can be found in heat transfer textbooks.

Smaller Value of *h* With Packaging

For packaged meat, the effective heat transfer coefficient *h* is reduced by two factors:

- the thermal resistance of the packaging.
- any trapped air layer between the package and the product.

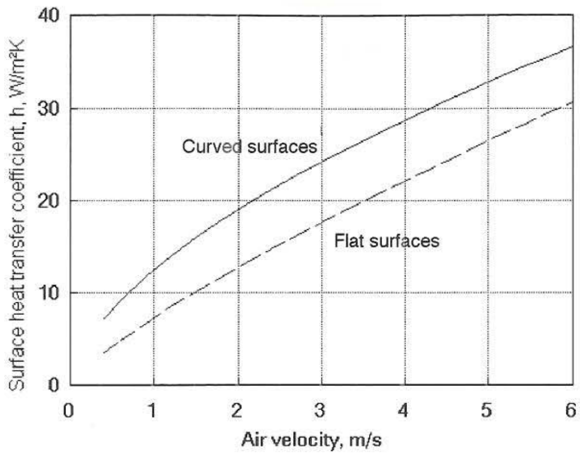

Figure 5. Estimated heat transfer coefficients for flat and curved surfaces at air velocities between 0.4 m/s and 6.0 m/s.

Although the packaging material can significantly affect heat transfer, often the most important factor is whether a layer of (still) air is trapped in the packaging.

As shown in Table 2, still air has a lower thermal conductivity than any of the packaging materials commonly used for meat. In fact, most methods of preventing heat transfer (such as household wall insulation, polystyrene panels and warm clothes) work primarily by trapping a layer of still air or gas within them.

Table 2. Typical thermal conductivities of packaging materials and still air. All values are in W/m K.

Solid cardboard	0.06 to 0.10
Plastic film	0.08 to 0.15
Corrugated cardboard	0.04 to 0.06
Still air	0.03

Many cartoned products often contain an air layer that is 0.5 to 2 mm thick around most of the product in the package, and a larger air gap on top.

In the case of stockinet and poly-bagged lamb carcasses, the layer of trapped air can reduce the effective heat transfer coefficient predicted by Figure 5 by 20% (for stockinet) or up to 50% (for loose polybags).

The overall effect of the packaging on the heat transfer coefficient can be calculated from:

$$\frac{1}{h} = \frac{1}{h_{air}} + \frac{x_{package}}{k_{package}} + \frac{x_{air}}{k_{air}}$$

Where:

h is the heat transfer coefficient to use in the calculations, W/m^2 K

h_{air} is the surface heat transfer coefficient read from Figure 5 or determined from the relevant equation (for cartoned product, the equation for flat surfaces; for carcasses, the equation for curved surfaces), W/m^2 K

$k_{package}$ is the thermal conductivity of the package from Table 2, W/m K

$x_{package}$ is the packaging material thickness, m

k_{air} is the thermal conductivity of still air from Table 2, W/m K

x_{air} is the thickness of the still air layer, m

Asymmetric Heat Transfer

The problem of asymmetric heat transfer arises frequently when predicting freezing time.

For example, the heat transfer coefficient at the top of a carton is often different from that at the bottom of the carton, due to the air gap that is intentionally left at the top to accommodate the meat as it freezes and expands. Figure 6 illustrates this situation.

There is a straightforward solution to this problem for slab-shaped food products, and this solution applies quite well to products that are almost slab-shaped too, such as meat cartons in plate or air-blast freezers.

When different h values apply at the top and bottom of a slab, the freezing fronts progress inwards at different rates. Instead of meeting in the geometric centre of the meat product, the freezing fronts will meet at a point $X_{actual}(1 - a)$ from the top and $X_{actual}(1 + a)$ from the bottom, where X_{actual} is the depth of the geometric centre for the actual product.

The value of a is calculated as follows:

The value of $r = \dfrac{h_{bottom}}{h_{top}}$ is needed,

where h_{bottom} and h_{top} are the heat transfer coefficients at the top and bottom of the carton respectively.

r is then used to calculate a, as follows:

$$a = \frac{r - 1}{r + 1 + 2Bi_{bottom}}$$

where Bi_{bottom} is known as the Biot number, and is given in this case by:

$$Bi_{bottom} = \frac{h_{bottom}\ X_{actual}}{k_f}$$

Having calculated a, we can then calculate the freezing time for the product using the freezing time equation but setting:

$h = h_{bottom}$ and

$X = (1 + a)\ X_{actual}$.

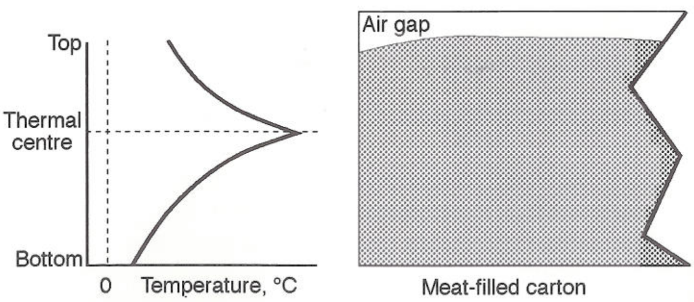

Figure 6. Temperature profiles in a chilling carton with an air gap at the top.

WORKED EXAMPLE

In this worked example, a sample problem is solved to demonstrate the method. Note that the intermediate calculation results shown here have been rounded; however, the full precision was used in the underlying calculations. If you repeat the calculations shown here and get slightly different answers, that is not a cause for concern.

Problem:
How long will it take for a 160 mm thick single-layer fibreboard carton of lean beef at 10°C to freeze to minus 18°C when placed in a freezer with air at 3 m/s and minus 30°C? There is an average air gap of 4 mm at the top of the carton.

To determine, the heat transfer coefficient at the top of the carton, the following equation is used:

$$\frac{1}{h_{top}} = \frac{1}{h_{air}} + \frac{x_{package}}{k_{package}} + \frac{x_{air}}{k_{air}}$$

For this equation, we need the following variables:

$k_{package}$ is the thermal conductivity of the package from Table 2, in this case, 0.06 W/mK

k_{air} is the thermal conductivity of still air from Table 2, in this case, 0.03

$x_{package}$ is the thickness of the fibreboard, in this case, 1.3 mm or 0.0013 m

x_{air} is the thickness of the still air layer, in this case, 4 mm or 0.004 m

Because the carton has a flat rather than a curved surface, the surface heat transfer coefficient, h_{air}, is determined by the equation,

$$h_{air} = 7.3\ v^{0.8}\ \text{W/m}^2\,\text{K}$$

Air velocity, v, is 3 m/s, so:

$$h_{air} = 7.3\ (3.0)^{0.8} = 17.6\ \text{W/m2 K}$$

Thus, the equation becomes

$$\frac{1}{h_{top}} = \frac{1}{17.6} + \frac{0.0013}{0.06} + \frac{0.004}{0.03}$$

So $h_{top} = 4.72\ \text{W/m}^2\,\text{K}$

Next, we need to determine the heat transfer coefficient for the bottom of the carton, h_{bottom}. The same equation is used. For the variables h_{air}, $k_{package}$, k_{air} and $x_{package}$, the values for the top and bottom of the carton will be the same. At the bottom of the carton, there is no layer of still air, so the value of x_{air} is 0.0.

Therefore the thermal conductivity at the bottom of the carton is affected only by the surface heat transfer coefficient (from Table 2) and the packaging material thermal conductivity and thickness:

$$\frac{1}{h_{bottom}} = \frac{1}{17.6} + \frac{0.0013}{0.06} + \frac{0}{0.03}$$

Therefore, the heat transfer coefficient of the bottom of the carton, h_{bottom}, = 12.73 W/m² K.

The distance to the physical thermal centre, X_{actual}, is needed. Since the meat is in a carton, X_{actual} is half the carton thickness. Therefore

$$X_{actual} = 80\ \text{mm or } 0.08\ \text{m.}$$

Next, we need to calculate the value of a, to be able to determine where the freezing fronts will meet. To do this, we need to know the value of r, where r is the ratio of the surface heat transfer coefficient on the top surface to that on the bottom surface.

$$r = \frac{h_{bottom}}{h_{top}}$$

$$\text{so } r = \frac{12.73}{4.72} = 2.70$$

and since:

$$Bi_{\text{bottom}} = \frac{h_{bottom}\ X_{actual}}{k_f} = \frac{12.73 \times 0.08}{1.21} = 0.84$$

then

$$a = \frac{r - 1}{r + 1 + 2Bi_{bottom}} = \frac{2.70 - 1}{2.70 + 1 + 2(0.84)} = 0.315$$

(Worked example continues on the back page)

The shortest distance to the thermal centre is

$$X = (1 + a)\, X_{actual}$$

so

$$X = (1 + 0.315) \times 0.08 = 0.105 \text{ m}$$

The mean freezing temperature of the meat, T_{fm}, *is*

$$T_{fm} = 1.8 + 0.263\, T_c + 0.105\, T_a$$

$$= 1.8 + 0.263\,(-18) + 0.105\,(-30) = -6.1°\text{C}$$

At this point, we need to know the thermal properties of the meat. We could obtain these from MIRINZ Publication No. 955 or other such sources, but in this case, we will use some typical thermal properties for lean meat. The latent heat of freezing, L, is calculated from:

$$L = H_f - C_f\,(T_f + 40)$$

$$= 316548 - 2260(-0.9 + 40) = 228182 \text{ J/kg}$$

To determine the freezing time, t_f, we need values for ΔH_1, ΔH_2, ΔT_1, and ΔT_2

$$\Delta H_1 = \rho_l\, C_l\,(T_{in} - T_{fm}) \quad = 1050 \times 3640 \times [(10 - (-6.1)] = 6.147\,(10^7) \text{ J/m}^3$$

$$\Delta H_2 = \rho_l\, L + \rho_f\, C_f\,(T_{fm} - T_c) \quad = 1050 \times 228182 + 950 \times 2260 \times [-6.1 - (-18)] = 2.651(10^8) \text{ J/kg}$$

$$\Delta T_1 = \frac{T_{in} - T_{fm}}{2} - T_a \quad = \frac{[10 - (-6.1)]}{2} - (-30) = 38.04°\text{C}$$

$$\Delta T_2 = T_{fm} - T_a \quad = -6.1 - (-30) = 23.9°\text{C}$$

From Table 1, the value of E, the Equivalent Heat Transfer Dimensionality, is 1.3, as the example is a beef carton being frozen in an air blast freezer.

Now that we have all the information that we need,

$$t_f = \frac{1}{E}\left(\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right)\left(\frac{X}{h_{bottom}} + \frac{X^2}{2k_f}\right)$$

so...

$$t_f = \frac{1}{1.3}\left(\frac{6.147(10^7)}{38.04} + \frac{2.651(10^8)}{23.9}\right) \times \left(\frac{0.105}{12.73} + \frac{0.105^2}{2 \times 1.21}\right)$$

$$= 125499 \text{ seconds or } 34.9 \text{ hours}$$

which is about what one would expect under these circumstances.

FURTHER READING

Willix, J. & Amos, N.D. (1995) *Thermal properties of foods*, MIRINZ Publ. No. 955. (Thermal properties of many food products, especially meat.)

Welty, J.R. (1978) *Engineering heat transfer - S.I. version*, Wiley, NY. (One of many good, general, heat transfer text books, although freezing time calculations are not specifically covered.)

Cleland, A.C. (1990) *Food refrigeration processes - analysis, design and simulation*, Elsevier Science Publishers, London. (Theoretical treatment of freezing time calculations by many methods, among other related issues.)

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